ALGEBRA FORMULAS

Factors and Zeros of Polynomials

Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_o$ be a polynomial. If $p(a) = 0$, then a is a *zero* of the polynomial and a solution of the equation $p(x) = 0$. Furthermore, $(x - a)$ is a factor of the polynomial.

Fundamental Theorem of Algebra

An *n*th degree polynomial has *n* (not necessarily distinct) zeros. Although all of these zeros may be imaginary, a real polynomial of odd degree must have at least one real zero.

Quadratic Formula

If $p(x) = ax^2 + bx + c$, and $0 \le b^2 - 4ac$, then the real zeros of p are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Special Factors

$x^2 - a^2 = (x - a)(x + a)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$

Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \dots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \dots \pm nxy^{n-1} \mp y^n$$

Rational Zero Theorem

If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_o$ has integer coefficients, then every *rational zero* of p is of the form $x = r/s$, where r is a factor of a_o and s is a factor of a_n.

Factoring by Grouping

$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$

Arithmetic Operations

$ab + ac = a(b + c)$	$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$	$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$
$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \frac{ad}{bc}$	$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$	$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$
$a\left(\frac{b}{c}\right) = \frac{ab}{c}$	$\frac{a - b}{c - d} = \frac{b - a}{d - c}$	$\frac{ab + ac}{a} = b + c$

Exponents and Radicals

$a = 1,\ a \neq 0$	$(ab)^x = a^x b^x$	$a^x a^y = a^{x+y}$	$\sqrt{a} = a^{1/2}$
$\sqrt[n]{a} = a^{1/n}$	$\sqrt[n]{a^m} = a^{m/n}$	$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$	$(a^x)^y = a^{xy}$
$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	$a^{-x} = \frac{1}{a^x}$	$\frac{a^x}{a^y} = a^{x-y}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$